Arizona Mathematic Standards

Mathematics Curriculum Map

Third Grade

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS

Chandler Unified School District Standards

Third Grade – At a Glance

Curriculum Map

***Use Perimeter, Area, and Quarter 4 standards as opening routines in Quarters 1-3 to ensure exposure to all standards prior to AZMerit.**

Quarter 1	Quarter 2	Quarter 3	Quarter 4
Number & Operations in Base Ten **Operations & Algebraic Thinking**	**Operations & Algebraic Thinking**	**Number & Operations - Fractions** Measurement and Data	**Measurement and Data** **Geometry**
3.NBT.A.1 3.OA.D.8 (Addition and Subtraction) 3.OA.D.9 (Addition) 3.MD.B.3 (Go Math) 3.MD.B.4 (Go Math) 3.OA.A.1 3.OA.A.3 3.OA.B.5 (Go Math) 3.NBT.A.2 (Fluency Standard) 3.OA.C.7 (Fluency Standard) 3.OA.D.10 – Embed all year	3.OA.A.2 3.OA.A.3 3.OA.A.4 3.OA.B.5 (Go Math) 3.OA.B.6 3.OA.C.7 3.OA.D.8 3.OA.D.9 3.NBT.A.3 3.NBT.A.2 (Fluency Standard) 3.OA.C.7 (Fluency Standard) 3.OA.D.10 – Embed all year	Continue with 3.OA.A.1-4 Continue with 3.OA.B.5-6 Continue with 3.OA.C.7 Continue with 3.OA.D.8-10 3.NF.A.1 3.NF.A.2 3.NF.A.3 3.MD.C.5 3.MD.C.6 3.MD.C.7 (Go Math) 3.MD.C.8 3.NBT.A.2 (Fluency Standard) 3.OA.C.7 (Fluency Standard) 3.OA.D.10 – Embed all year	3.MD.A.1a 3.MD.A.1b 3.MD.A.2 3.MD.B.3 (My Math) 3.MD.B.4 3.MD.C.5 (Go Math) 3.MD.C.7 3.MD.C.8 (Go Math) 3.G.A.1 3.G.A.2 3.NBT.A.2 (Fluency Standard) 3.OA.C.7 (Fluency Standard) 3.OA.D.10 – Embed all year

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

All quarters will include the Mathematical Practices

NOTE: Mathematical practices are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics.

Grade 3 Key: NBT = Number and Operations in Base Ten, OA = Operations and Algebraic Thinking, NF = Number and Operations – Fraction, MD = Measurement and Data, G = Geometry

Third Grade Overview

Operations and Algebraic Thinking (OA)
Note: Grade 3 expectations in this domain are limited to multiplication through 10 x 10 and division with both quotients and divisors less than or equal to 10.

- Represent and solve problems involving whole number multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations in Base Ten (NBT)
Note: A range of algorithms may be used.

- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions (NF)
Note: Grade 3 expectations are limited to fractions with denominators: 2, 3, 4, 6, 8

- Understand fractions as numbers.

Measurement and Data (MD)

- Solve problems involving measurement.
- Represent and interpret data.
- Geometric measurement: Understand concepts of area and perimeter.

Geometry (G)
- Reason with shapes and their attributes.

Standards for Mathematical Practices (MP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Third Grade Content Emphasis

Third Grade Content Emphasis	
Operations and Algebraic Thinking (OA)	
●	Represent and solve problems involving whole number multiplication and division.
●	Understand properties of multiplication and the relationship between multiplication and division.
●	Multiply and divide within 100.
●	Solve problems involving the four operations, and identify and explain patterns in arithmetic.
Number and Operations in Base Ten (NBT)	
▲	Use place value understanding and properties of operations to perform multi-digit arithmetic.
Number and Operations – Fractions (NF)	
●	Understand fractions as numbers.
Measurement and Data (MD)	
▲	Solve problems involving measurement.
▲	Represent and interpret data.
●	Geometric measurement: Understand concepts of area and perimeter.
Geometry (G)	
▲	Reason with shapes and their attributes.
● -Major Content ▲ -Supporting Content	
Major content (●) from the content emphasis section should account for approximately 70% of instructional time.	

Quarter 1

Essential Question(s):

- **How can numbers be expressed, ordered, and compared?**
- **How does place value help me add larger numbers?**
- **How are the operations of subtraction and addition related?**
- **What does multiplication mean?**

80% of instructional time should be based on core materials (My Math, Go Math, Saxon); 20% may be based on additional resources.

Topic	Arizona Mathematics Standards	Resources
Fluency Standards	*To be taught throughout the year:* 3.NBT.A.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. 3.OA.C.7 Fluently multiply and divide within 100. By the end of Grade 3, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10.	
Place Value	3.NBT.A.1 Use place value understanding to round whole numbers to the nearest 10 or 100.	My Math Chapter 1 **Additional Resources:** Illustrative Mathematics 3.NBT.A.1 Tasks Georgia Standards – Unit 1 Inside Mathematics 3rd Grade NBT Tasks
Addition	3.NBT.A.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. 3.OA.D.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Utilize understanding of the Order of Operations when there are no parentheses. (**Addition**) 3.OA.D.9 Identify patterns in the **addition table** and the multiplication table and explain them using properties of operations (e.g. observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends).	My Math Chapter 2 **Additional Resources:** EngageNY Module 2 Topic D Illustrative Mathematics 3.NBT.A.2 Tasks 3.OA.D.8 Tasks 3.OA.D.9 Tasks Inside Mathematics 3rd Grade OA Tasks

Quarter 1

Essential Question(s):

- **How can numbers be expressed, ordered, and compared?**
- **How does place value help me add larger numbers?**
- **How are the operations of subtraction and addition related?**
- **What does multiplication mean?**

Subtraction	3.NBT.A.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. 3.OA.D.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Utilize understanding of the Order of Operations when there are no parentheses. (**Addition and Subtraction**) 3.OA.D.10 When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding.	My Math Chapter 3 **Additional Resources:** EngageNY Module 2 Topic E Illustrative Mathematics 3.NBT.A.2 Tasks 3.OA.D.8 Tasks 3.OA.D.9 Tasks Inside Mathematics 3rd Grade OA Tasks
Understand Multiplication	3.OA.A.1 Interpret products of whole numbers as the total number of objects in equal groups (e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each). 3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities. *See Table 2*. 3.OA.D.10 When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding.	My Math Chapter 4 **Additional Resources:** EngageNY Module 1 Topic A Topic C Illustrative Mathematics 3.OA.A.3 Tasks Inside Mathematics 3rd Grade OA Tasks

Quarter 1

Essential Question(s):

- **How can numbers be expressed, ordered, and compared?**
- **How does place value help me add larger numbers?**
- **How are the operations of subtraction and addition related?**
- **What does multiplication mean?**

Understand Division	3.OA.A.2 Interpret whole number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each group when 56 objects are partitioned equally into 8 groups, or as a number of groups when 56 objects are partitioned into equal groups of 8 objects each). *See Table 2.* 3.OA.C.7 Fluently multiply and divide within 100. By the end of Grade 3, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10. 3.OA.D.10 When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding.	My Math Chapter 5 **Additional Resources:** EngageNy Module 1 Topic D Illustrative Mathematics 3.OA.A.2 Tasks 3.OA.C.7 Tasks Georgia Standards – Unit 2 Inside Mathematics 3rd Grade OA Tasks

***The Standards for Mathematical Practices are to be embedded throughout every chapter.**

Quarter 2

Essential Question(s):

- **What does multiplication mean?**
- **What does division mean?**
- **What is the importance of patters in learning multiplication and division?**
- **What strategies can be used to learn multiplication and division facts?**

80% of instructional time should be based on core materials (My Math, Go Math, Saxon); 20% may be based on additional resources.

Topic	Arizona Mathematics Standards	Resources
Fluency Standards	*To be taught throughout the year:* 3.NBT.A.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. 3.OA.C.7 Fluently multiply and divide within 100. By the end of Grade 3, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10.	
*Standard to Supplement throughout Quarter 2	3.OA.B.6 Understand division as an unknown-factor problem (e.g., find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8).	Supplemental Resources: EngageNy Module 1 Topic B Georgia Standards – Unit 2 Inside Mathematics 3rd Grade OA Tasks
Multiplication and Division Patterns	3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities. *See Table 2*. 3.OA.A.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers *For example, determine the unknown number that makes the equation true in each of the equations 8 x □ = 48, 5 = □ ÷ 3, 6 x 6 = □ . See Table 2*. 3.OA.C.7 Fluently multiply and divide within 100. By the end of Grade 3, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10. 3.OA.D.9 Identify patterns in the addition table and the multiplication table and explain them using properties of operations (e.g. observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends). 3.NBT.A.3 Multiply one-digit whole numbers by multiples of 10 in the range 10 to 90 using strategies based on place value and the properties of operations (e.g., 9 x 80, 5 x 60).	My Math Chapter 6 **Additional Resources:** Illustrative Mathematics 3.OA.A.3 Tasks 3.OA.A.4 Tasks 3.OA.C.7 Tasks 3.OA.D.9 Tasks 3.NBT.A.3 Tasks Georgia Standards – Unit 2 Inside Mathematics 3rd Grade OA Tasks 3rd Grade NBT Tasks

Quarter 2

Essential Question(s):

- **What does multiplication mean?**
- **What does division mean?**
- **What is the importance of patters in learning multiplication and division?**
- **What strategies can be used to learn multiplication and division facts?**

Multiplication and Division	3.OA.A.2 Interpret whole number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each group when 56 objects are partitioned equally into 8 groups, or as a number of groups when 56 objects are partitioned into equal groups of 8 objects each). *See Table 2.* 3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities. *See Table 2*. 3.OA.A.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers *For example, determine the unknown number that makes the equation true in each of the equations 8 x □ = 48, 5 = □ ÷ 3, 6 x 6 = □ . See Table 2*. 3.OA.D.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Utilize understanding of the Order of Operations when there are no parentheses. 3.OA.D.9 Identify patterns in the addition table and the multiplication table and explain them using properties of operations (e.g. observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends). 3.OA.D.10 When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding.	My Math Chapter 7 **Additional Resources:** EngageNY Module 1 Topic E Module 3 Topic B Topic C Topic D Illustrative Mathematics 3.OA.A.2 Tasks 3.OA.A.3 Tasks 3.OA.A.4 Tasks 3.OA.D.8 Tasks 3.OA.D.9 Tasks Georgia Standards – Unit 2 Inside Mathematics 3rd Grade OA Tasks

***The Standards for Mathematical Practices are to be embedded throughout every chapter.**

Quarter 3

Essential Question(s):

- **How can multiplication and division facts with small numbers be applied to larger numbers?**
- **How are properties and equations used to group numbers?**
- **How can fractions be used to represent numbers and their parts?**
- **How are perimeter and area related and how are they different?**

80% of instructional time should be based on core materials (My Math, Go Math, Saxon); 20% may be based on additional resources.

Topic	Arizona Mathematics Standards	Resources
Fluency Standards	*To be taught throughout the year:* 3.NBT.A.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. 3.OA.C.7 Fluently multiply and divide within 100. By the end of Grade 3, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10.	
Apply Multiplication and Division	3.OA.A.1 Interpret products of whole numbers as the total number of objects in equal groups (e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each). 3.OA.A.2 Interpret whole number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each group when 56 objects are partitioned equally into 8 groups, or as a number of groups when 56 objects are partitioned into equal groups of 8 objects each). *See Table 2.* 3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities. *See Table 2*. 3.OA.C.7 Fluently multiply and divide within 100. By the end of Grade 3, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10. 3.OA.D.9 Identify patterns in the addition table and the multiplication table and explain them using properties of operations (e.g. observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends). 3.OA.D.10 When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding.	My Math Chapter 8 **Additional Resources:** Illustrative Mathematics 3.OA.A.2 Tasks 3.OA.A.3 Tasks 3.OA.C.7 Tasks 3.OA.D.9 Tasks Georgia Standards – Unit 2 Inside Mathematics 3rd Grade OA Tasks

Quarter 3

Essential Question(s):

- **How can multiplication and division facts with small numbers be applied to larger numbers?**
- **How are properties and equations used to group numbers?**
- **How can fractions be used to represent numbers and their parts?**
- **How are perimeter and area related and how are they different?**

Properties and Equations	3.OA.B.5 Apply properties of operations as strategies to multiply and divide. Properties include commutative and associative properties of multiplication and the distributive property. (Students do not need to use the formal terms for these properties.) 3.OA.D.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Utilize understanding of the Order of Operations when there are no parentheses.	My Math Chapter 9 **Additional Resources:** EngageNY Module 1 Topic F Module 3 Topic A Illustrative Mathematics 3.OA.B.5 Tasks 3.OA.D.8 Tasks Inside Mathematics 3rd Grade OA Tasks
Fractions	3.NF.A.1 Understand a fraction (1/*b*) as the quantity formed by one part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by *a* parts of size 1/*b*. 3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Understand that each part has size 1/*b* and that the end point of the part based at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a/b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Understand that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line including values greater than 1. c. Understand a fraction 1/*b* as a special type of fraction that can be referred to as a unit fraction (e.g. 1/2, 1/4).	My Math Chapter 10 **Additional Resources:** EngageNY Module 5 Illustrative Mathematics 3.NF.A.1 Tasks 3.NF.A.2 Tasks 3.NF.A.3 Tasks Georgia Standards – Unit 5 Inside Mathematics 3rd Grade NF Tasks

Quarter 3

Essential Question(s):

- **How can multiplication and division facts with small numbers be applied to larger numbers?**
- **How are properties and equations used to group numbers?**
- **How can fractions be used to represent numbers and their parts?**
- **How are perimeter and area related and how are they different?**

Fractions Continued	3.NF.A.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent if they have the same relative size compared to 1 whole. b. Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Understand that comparisons are valid only when the two fractions refer to the same whole. Record results of comparisons with the symbols >, =, or <, and justify conclusions.	
Perimeter and Area	3.MD.C.5 Understand area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. 3.MD.C.8 Solve real-world and mathematical problems involving perimeters of plane figures and areas of rectangles, including finding the perimeter given the side lengths, finding an unknown side length. Represent rectangles with the same perimeter and different areas or with the same area and different perimeters.	My Math Chapter 13 Lessons 1-4 **Additional Resources:** Inside Mathematics 3rd Grade MD Tasks
*Standard to Supplement with Chapter 13	3.MD.C.6 Measure areas by counting unit squares (e.g., square cm, square m, square in, square ft, and improvised units).	Supplemental Resources: EngageNY Module 4 Topic A Topic C Illustrative Mathematics 3.MD.C.6 Tasks Inside Mathematics 3rd Grade MD Tasks

***The Standards for Mathematical Practices are to be embedded throughout every chapter.**

Quarter 4

Essential Question(s):

- **Why do we measure?**
- **How do we obtain useful information from a set of data?**
- **How can geometric shapes help me solve real-world problems?**

80% of instructional time should be based on core materials (My Math, Go Math, Saxon); 20% may be based on additional resources.

Topic	Arizona Mathematics Standards	Resources
Fluency Standards	*To be taught throughout the year:* 3.NBT.A.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. 3.OA.C.7 Fluently multiply and divide within 100. By the end of Grade 3, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10.	
Perimeter and Area	3.MD.C.7 Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Understand that rectilinear figures can be decomposed into non-overlapping rectangles and that the sum of the areas of these rectangles is identical to the area of the original rectilinear figure. Apply this technique to solve problems in real-world contexts. 3.OA.D.10 When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding.	My Math Chapter 13 Lessons 5-10 **Additional Resources:** EngageNY Module 7 Topic C Topic D Topic E Illustrative Mathematics 3.MD.C.7 Tasks

Quarter 4

Essential Question(s):

- **Why do we measure?**
- **How do we obtain useful information from a set of data?**
- **How can geometric shapes help me solve real-world problems?**

Measurement	3.MD.A.1a Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes (e.g., representing the problem on a number line diagram). 3.MD.A.2 Measure and estimate liquid volumes and masses of objects using metric units. (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units. Excludes multiplicative comparison problems (problems involving notions of "times as much"). See Table 2.	My Math Chapter 11 **Additional Resources:** EngageNY Module 2 Topic A Topic B Topic C Illustrative Mathematics 3.MD.A.1 Tasks 3.MD.A.2 Tasks Georgia Standards – Unit 6 Inside Mathematics 3rd Grade MD Tasks
Represent and Interpret Data	3.MD.B.3 Create a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *See Table 1.* 3.MD.B.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch to the nearest quarter-inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.	My Math Chapter 12 **Additional Resources:** EngageNy Module 6 Topic A Topic B Illustrative Mathematics 3.MD.B.3 Tasks Georgia Standards – Unit 6 Inside Mathematics 3rd Grade MD Tasks

Quarter 4

Essential Question(s):

- **Why do we measure?**
- **How do we obtain useful information from a set of data?**
- **How can geometric shapes help me solve real-world problems?**

Geometry	3.G.A.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples quadrilaterals that do not belong to any of these subcategories. 3.G.A.2 Partition shapes into *b* parts with equal areas. Express the area of each part as a unit fraction 1/*b* of the whole. (Grade 3 expectations are limited to fractions with denominators *b* = 2,3,4,6,8.)	My Math Chapter 14 **Additional Resources:** EngageNY Module 7 Topic A Topic B Illustrative Mathematics 3.G.A.2 Tasks Georgia Standards – Unit 4 Inside Mathematics 3rd Grade G Tasks
*Standard to Supplement	3.MD.A.1b Solve word problems involving money through $20.00, using symbols $, ".", ¢.	

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The Mathematical Practices: Narratives and Questions

Mathematics Practices		Narratives	Related Questions
Overarching habits of mind of a productive math thinker	**4.MP.1 Make sense of problems and persevere in solving them**	Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.	• How would you describe the problem in your own words? • How would you describe what you are trying to find? • What do you notice about...? • What information is given in the problem? • Describe the relationship between the quantities. • Describe what you have already tried. What might you change? • Talk me through the steps you’ve used to this point. • What steps in the process are you most confident about? • What are some other strategies you might try? • What are some other problems that are similar to this one? • How might you use one of your previous problems to help you begin? • How else might you organize...represent... show...?
	4.MP.6 Attend to precision	Mathematically proficient students clearly communicate to others using appropriate mathematical terminology, and craft explanations that convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.	• What mathematical terms apply in this situation? • How did you know your solution was reasonable? • Explain how you might show that your solution answers the problem. • What would be a more efficient strategy? • How are you showing the meaning of the quantities? • What symbols or mathematical notations are important in this problem? • What mathematical language...,definitions..., properties can you use to explain...? • How could you test your solution to see if it answers the problem?

Actions and dispositions from NCSM Summer Leadership Academy, Atlanta, GA • Draft, June 22, 2011)
Most questions from all Grades Common Core State Standards Flip Book

The Mathematical Practices: Narratives and Questions

Mathematics Practices		Narratives	Related Questions
Reasoning and Explaining	**4.MP.2 Reason abstractly and quantitatively**	Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.	• What do the numbers used in the problem represent? • What is the relationship of the quantities? • How is _____ related to _____? • What is the relationship between _____and _____? • What does_____mean to you? (e.g. symbol, quantity, diagram) • What properties might we use to find a solution? • How did you decide in this task that you needed to use...? • Could we have used another operation or property to solve this task? Why or why not?
	4.MP.3 Construct viable arguments and critique the reasoning of others	Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming or questioning the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.	• What mathematical evidence would support your solution? • How can we be sure that...? / How could you prove that...? • Will it still work if...? • What were you considering when...? • How did you decide to try that strategy? • How did you test whether your approach worked? • How did you decide what the problem was asking you to find? • Did you try a method that did not work? Why didn't it work? Could it work? • What is the same and what is different about...? • How could you demonstrate a counter-example?

Actions and dispositions from NCSM Summer Leadership Academy, Atlanta, GA • Draft, June 22, 2011)
Most questions from all Grades Common Core State Standards Flip Book

The Mathematical Practices: Narratives and Questions

Mathematics Practices		Narratives	Related Questions
Modeling and Using Tools	**4.MP.4 Model with mathematics**	Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	• What number model could you construct to represent the problem? • What are some ways to represent the quantities? • What is an equation or expression that matches the diagram, number line, chart, table, and your actions with the manipulatives? • Where did you see one of the quantities in the task in your equation or expression? What does each number in the equation mean? • How would it help to create a diagram, graph, table...? • What are some ways to visually represent...? • What formula might apply in this situation?
	4.MP.5 Use appropriate tools strategically	Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.	• What mathematical tools can we use to visualize and represent the situation? • Which tool is more efficient? Why do you think so? • What information do you have? • What do you know that is not stated in the problem? • What approach are you considering trying first? • What estimate did you make for the solution? • In this situation would it be helpful to use...a graph..., number line..., ruler..., diagram..., calculator..., manipulative? • Why was it helpful to use...? • What can using a _____ show us that _____may not? • In what situations might it be more informative or helpful to use...?

Actions and dispositions from NCSM Summer Leadership Academy, Atlanta, GA • Draft, June 22, 2011)
Most questions from all Grades Common Core State Standards Flip Book

The Mathematical Practices: Narratives and Questions

Mathematics Practices		Narratives	Related Questions
Seeing structure and generalizing	**4.MP.7 Look for and make use of structure**	Mathematically proficient students use structure and patterns to assist in making connections among mathematical ideas or concepts when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.	• What observations do you make about...? • What do you notice when...? • What parts of the problem might you eliminate..., simplify...? • What patterns do you find in...? • How do you know if something is a pattern? • What ideas that we have learned before were useful in solving this problem? • What are some other problems that are similar to this one? • How does this relate to...? • In what ways does this problem connect to other mathematical concepts?
	4.MP.8 Look for and express regularity in repeated reasoning	Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.	• Explain how this strategy works in other situations? • Is this always true, sometimes true or never true? • How would we prove that...? • What do you notice about...? • What is happening in this situation? • What would happen if...? • Is there a mathematical rule for...? • What predictions or generalizations can this pattern support? • What mathematical consistencies do you notice?

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Table 1. Common Addition and Subtraction Problem Types/Situations. [1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put together/Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?”): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).
[2] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children that the = sign does not always mean **makes** or **results in** but always does mean **is the same quantity as.**
[3] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

Table 2. Common Multiplication and Division Situations. [7]

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	3 x 6 = ?	**3 x ? = 18, and 18 ÷ 3 = ?**	**? x 6 = 18, and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurements example:* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example:* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example:* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays [4]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example:* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example:* A rectangle has the area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example:* A rectangle has the area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare (Grade 4 ONLY)	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example:* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example:* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example:* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	a x b = ?	a x ? = p, and p ÷ a = ?	? x b = p, and p ÷ b = ?

[7] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[4] The language is the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.